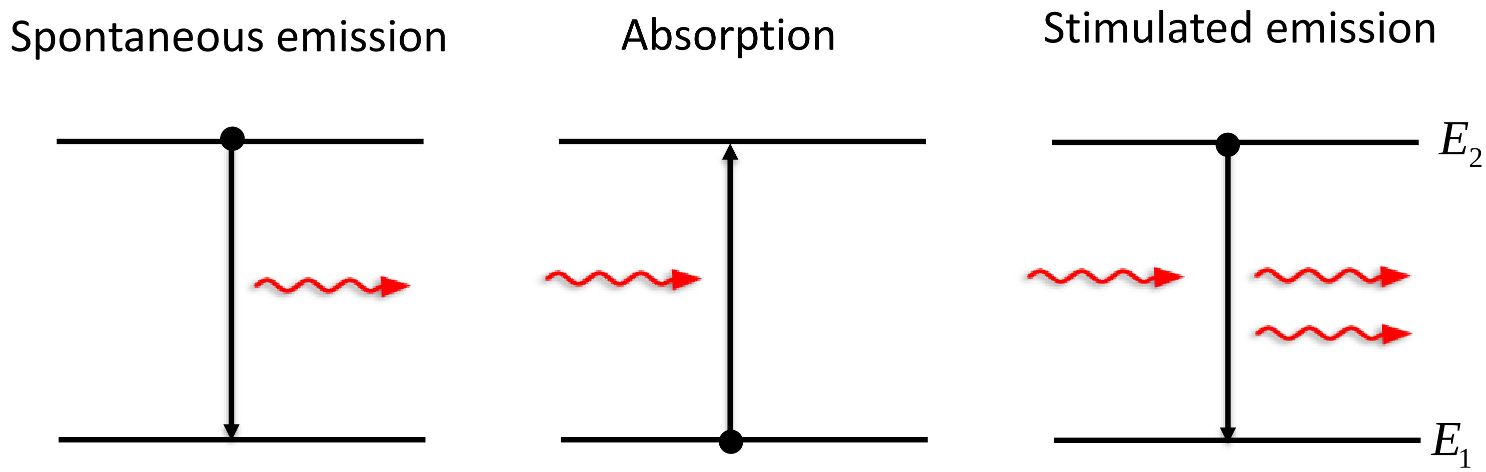
EE 232: Lightwave Devices
Lecture #12 – Spontaneous emission

Instructor: Seth A. Fortuna

Dept. of Electrical Engineering and Computer Sciences
University of California, Berkeley

2/28/2019

Two-level system



$A_{21} =$ Spontaneous emission rate

$B_{21}\rho(\hbar\omega) =$ Stimulated emission rate

$B_{12}\rho(\hbar\omega) =$ Absorption rate

$\rho(\hbar\omega) =$ Spectral photon density

$$\frac{dN_2}{dt} = -A_{21}N_2 - B_{21}\rho(\hbar\omega)N_2 + B_{12}\rho(\hbar\omega)N_1$$

$$\frac{dN_2}{dt} = 0$$ (steady state)

$$\rightarrow \rho(\hbar\omega) = \frac{A_{21}/B_{21}}{(N_1/N_2)(B_{12}/B_{21}) - 1}$$

Photon mode density

We place our two-level system in a "cavity"
What are the density of photon modes in this cavity?

$$\text{Mode density} = \frac{1}{V}\sum_k$$

$$= \frac{1}{V}\int \frac{2d^3\mathbf{k}}{(2\pi)^3/V}$$ (factor of 2 accounts for polarization)

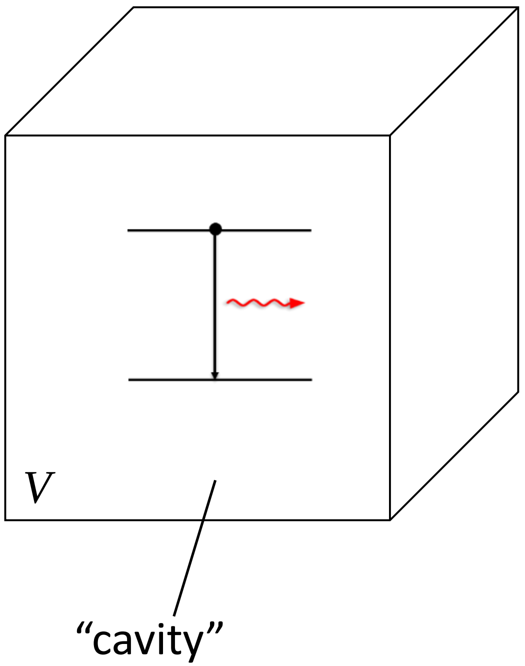
$$= \int \frac{8\pi k^2 dk}{(2\pi)^3}$$

$$= \int \frac{k^2 dk}{\pi^2} \qquad \omega = \frac{c}{n}k \qquad \frac{d\omega}{dk} = \frac{c}{n}$$

$$= \int \frac{\omega^2/(c/n)^2}{\pi^2\hbar}\frac{d(\hbar\omega)}{c/n}$$

$$= \int \frac{\omega^2 n^3}{\pi^2\hbar c^3} d(\hbar\omega)$$

$$= \int \rho_{mode}\, d(\hbar\omega)$$



$$\rho_{mode} = \frac{\omega^2 n^3}{\pi^2\hbar c^3}$$ (modes/cm³/eV)

Relations between the coefficients

$$n_{ph}(\hbar\omega) = \frac{1}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1}$$

Bose-Einstein distribution (photons per state)

$$\rho(\hbar\omega) = \rho_{mode}(\hbar\omega) n_{ph}(\hbar\omega)$$

$$\rho(\hbar\omega) = \frac{\omega^2 n^3}{\pi^2 \hbar c^3} \frac{1}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1}$$

Recall, we also derived

$$\rho(\hbar\omega) = \frac{A_{21} / B_{21}}{(N_1 / N_2)(B_{12} / B_{21}) - 1}$$

These equations must be equal to each other. This is only possible if:

$$N_1 / N_2 = \exp\left(\frac{\hbar\omega}{kT}\right)$$

$$B_{12} = B_{21}$$

$$A_{21} / B_{21} = \rho_{mode}(\hbar\omega)$$

Spontaneous emission

Transition rate from State 2 to State 1

$$R_{2\to 1} = A_{21}N_2 + B_{21}\rho(\hbar\omega)N_2$$

$$= B_{21}\rho_{mode}(\hbar\omega)N_2 + B_{21}\rho_{mode}(\hbar\omega)n_{ph}(\hbar\omega)N_2$$

$$= N_2 B_{21}\rho_{mode}(\hbar\omega)\left[1 + n_{ph}(\hbar\omega)\right]$$

- 1: stimulated emission from **one** photon in each mode
- n_{ph}: stimulated emission from $\boldsymbol{n_{ph}}$ photons in each mode

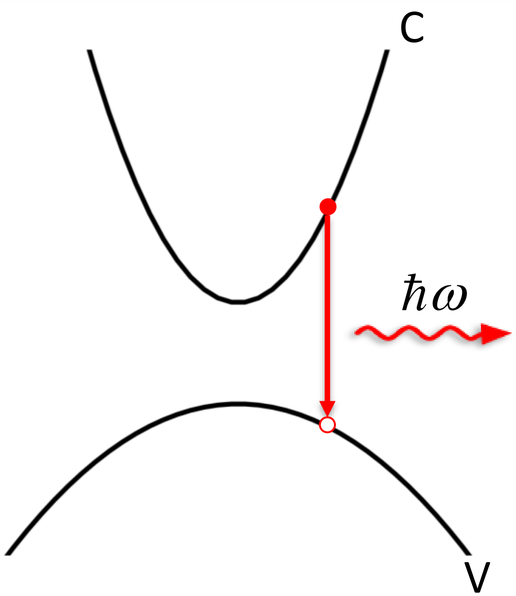
Spontaneous emission can be interpreted as the stimulated emission from a single photon in each optical mode.

Spontaneous emission in semiconductor

$$r_{spon} = \frac{2}{V}\sum_{k_c}\sum_{k_v}\frac{2\pi}{\hbar}\left|H_{cv}\right|^2 \delta(E_e - E_h - \hbar\omega) f_c(1 - f_v)$$

$$\left|H_{cv}\right|^2 = \left|\langle\psi_c\right| - \frac{qA_0}{2m_0}\hat{e}\cdot\mathbf{p}\left|\psi_v\rangle\right|^2$$

We need to relate the magnitude of the vector potential to the mode density.



$$\text{EM energy density} = \frac{\rho_{mode}(\hbar\omega)(1)V\hbar\omega}{V} = \rho_{mode}(\hbar\omega)\hbar\omega$$

$$\text{EM energy density} = \frac{1}{2}\epsilon_0 n^2 E_0^2 = \frac{1}{2}\epsilon_0 n^2 \omega^2 A_0^2$$

$$A_0^2 = \frac{2\rho_{mode}(\hbar\omega)\hbar\omega}{\epsilon_0 n^2 \omega^2 \hbar} = \frac{2n\omega}{\epsilon\pi^2 c^3}$$

Spontaneous emission in semiconductor

$$r_{spon}(\hbar\omega) = \frac{2}{V}\sum_{k_c}\sum_{k_v}\frac{2\pi}{\hbar}\left|H_{cv}\right|^2 \delta(E_e - E_h - \hbar\omega) f_c(1 - f_v)$$

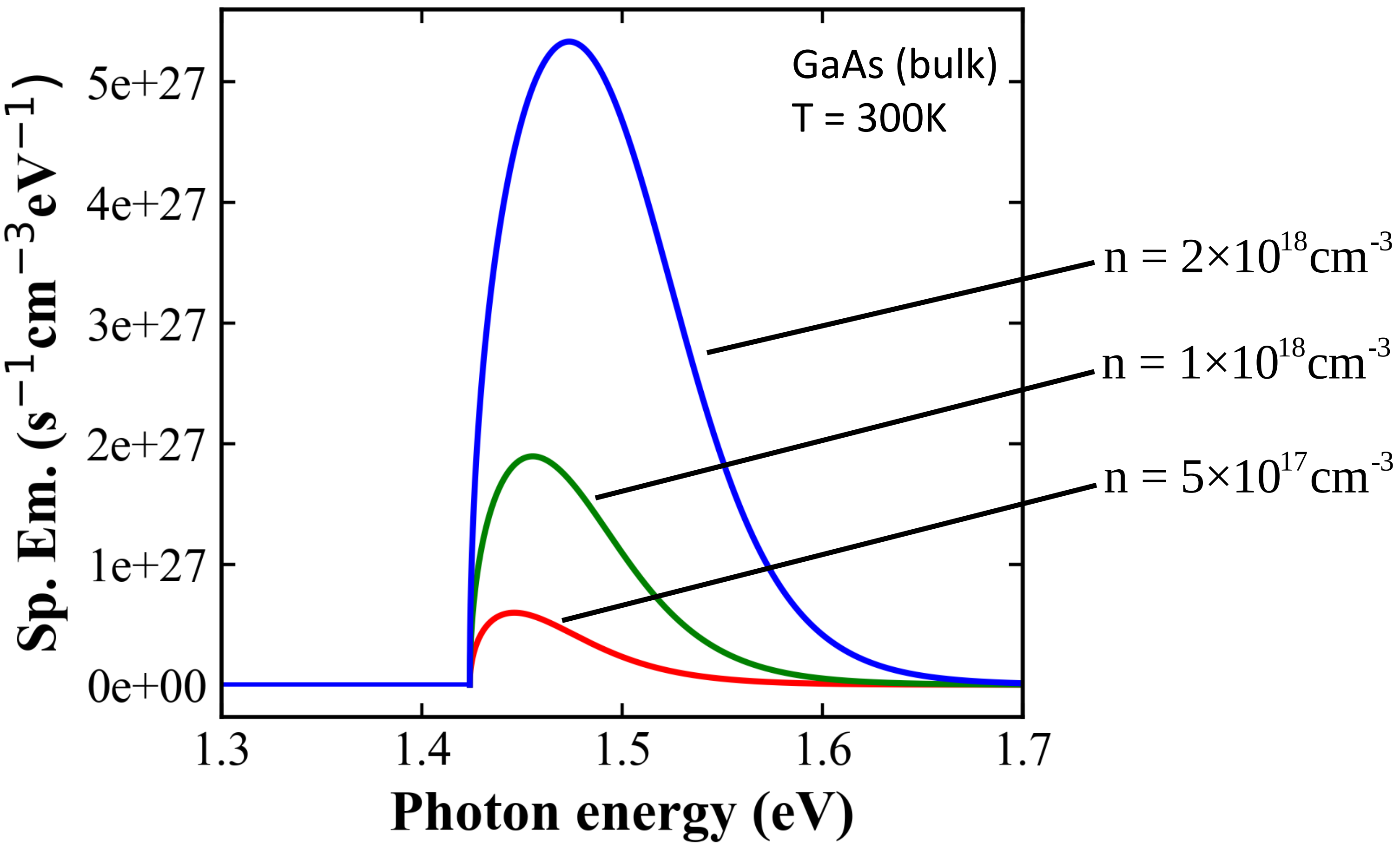
$$= \frac{2\pi}{\hbar}\frac{2}{V}\frac{q^2}{(2m_0)^2}\frac{2n\hbar\omega}{\epsilon_0\pi^2 c^3}\sum_{k_c}\sum_{k_v}\left|H_{cv}\right|^2 \delta(E_e - E_h - \hbar\omega) f_c(1 - f_v)$$

$$= C_0\left(\frac{n^2\omega^2}{\pi^2\hbar c^2}\right)\sum_{k_c}\sum_{k_v}\left|\hat{e}\cdot\mathbf{p}_{cv}\right|^2 \delta(E_e - E_h - \hbar\omega) f_c(1 - f_v)$$

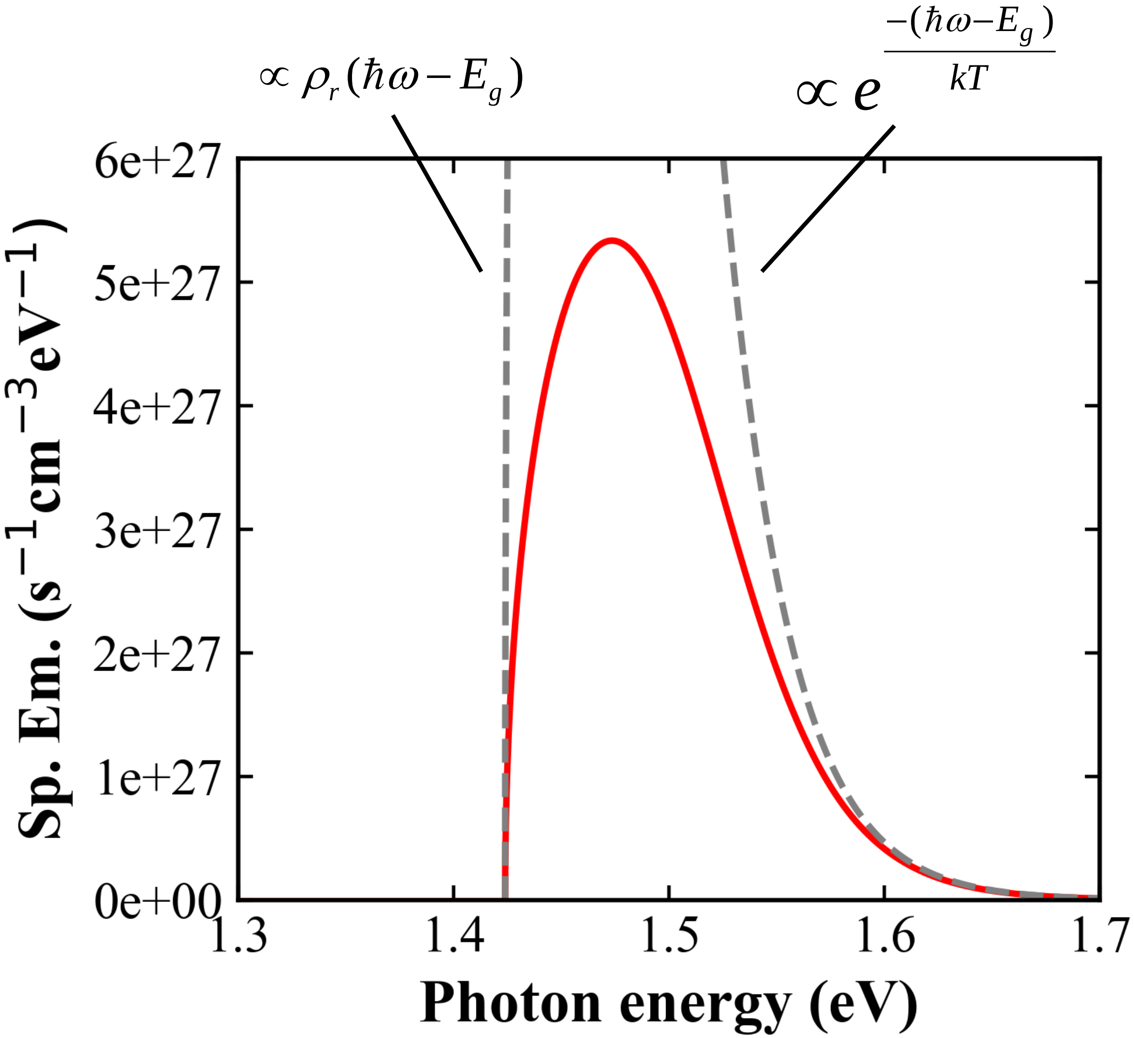
Spontaneous emission spectrum $\mathrm{cm^3 s^{-1} eV^{-1}}$

$$r_{spon,bulk}(\hbar\omega) = C_0\left(\frac{n^2\omega^2}{\pi^2\hbar c^2}\right)\left|\hat{e}\cdot\mathbf{p}_{cv}\right|^2 \rho_{r,bulk}(\hbar\omega - E_g) f_c(\hbar\omega - E_g)(1 - f_v(\hbar\omega - E_g))$$

Spontaneous emission spectrum



Spontaneous emission spectrum



For $\hbar\omega \approx E_g$

Limited by density of states

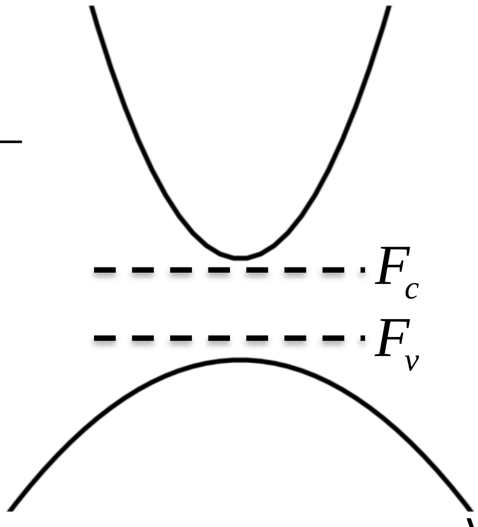
$$r_{spon} \propto \rho_r(\hbar\omega - E_g)$$

For $\hbar\omega \gg F_c - F_v$

Limited by filling of states

$$r_{spon} \propto e^{\frac{-(\hbar\omega - E_g)}{kT}}$$

Low-injection



For low-injection (Quasi-Fermi levels are within the bandgap)

$$f_c \simeq \exp\left(\frac{F_c - E_g - (\hbar\omega - E_g)(m_r^*/m_e^*)}{kT}\right)$$

$$(1 - f_v) \sim \exp\left(\frac{-(\hbar\omega - E_g)(m_r^*/m_h^*) - F_v}{kT}\right)$$

$$f_c(1 - f_v) \simeq \exp\left(\frac{F_c - E_g - (\hbar\omega - E_g)(m_r^*/m_e^*) - (\hbar\omega - E_g)(m_r^*/m_h^*) - F_v}{kT}\right)$$

$$= \exp\left(\frac{-(\hbar\omega - E_g)}{kT}\right)\exp\left(\frac{-E_g}{kT}\right)\exp\left(\frac{F_c - F_v}{kT}\right)$$

$$= \exp\left(\frac{-(\hbar\omega - E_g)}{kT}\right)\frac{np}{N_c N_v}$$

N_c, N_v (effective density of states)

Radiative rate

$$r_{spon,bulk}(\hbar\omega) = C_0 \left(\frac{n^2\omega^2}{\pi^2\hbar c^2} \right) \left| \hat{e} \cdot \mathbf{p}_{cv} \right|^2 \rho_{r,bulk}(\hbar\omega - E_g) \exp\left(\frac{-(\hbar\omega - E_g)}{kT} \right) \frac{np}{N_c N_v}$$

Radiative rate

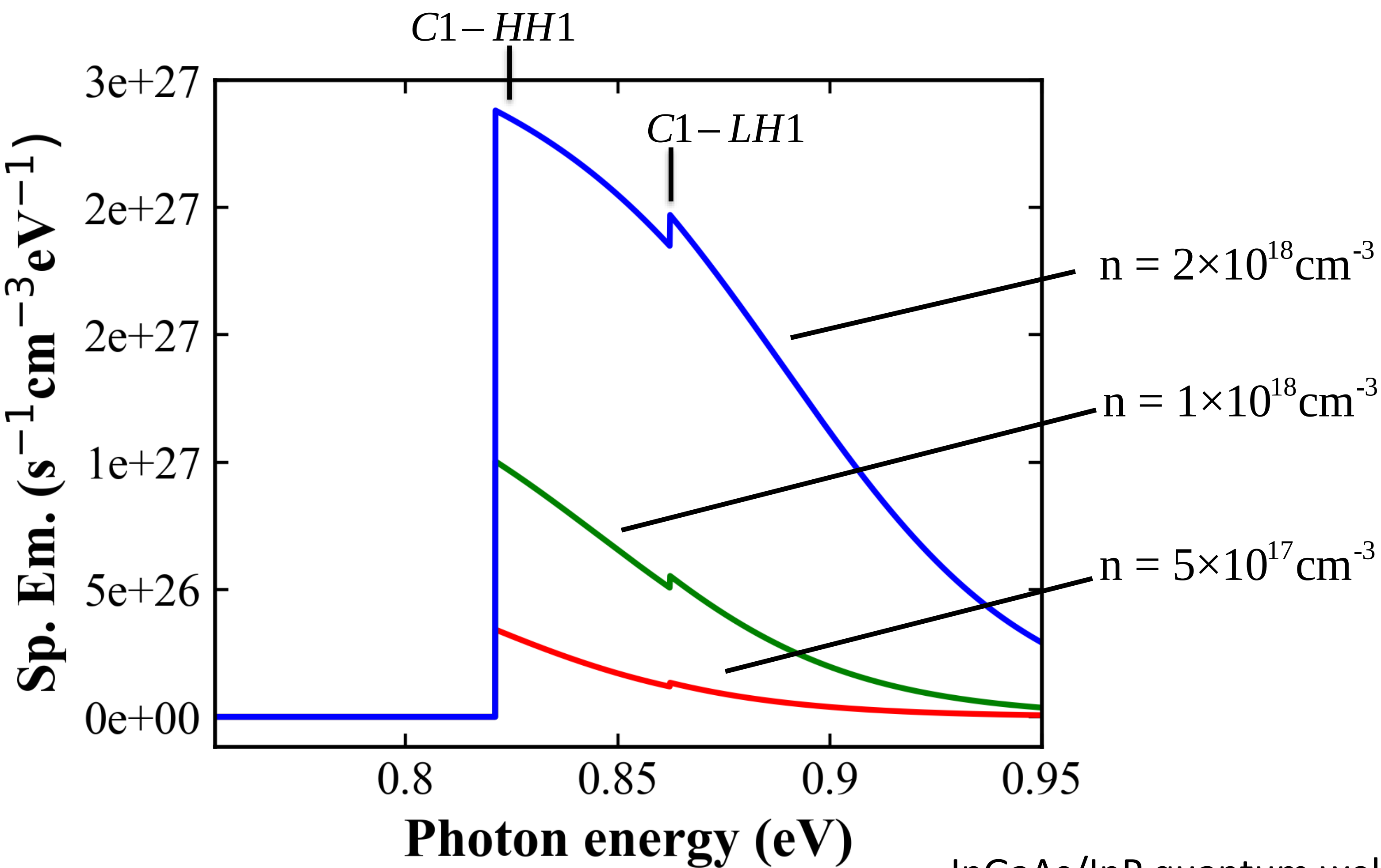
$cm^{-3}s^{-1}$

$$R_{rad} = \int r_{spon,bulk}(\hbar\omega) d(\hbar\omega) = B_0 np$$

Material	B_0 (cm^3s^{-1})
GaAs	2.0×10^{-10}
InP	1.2×10^{-10}
GaN	2.2×10^{-10}
GaP	3.9×10^{-13}
Si	3.2×10^{-14}
Ge	2.8×10^{-13}

Quantum well

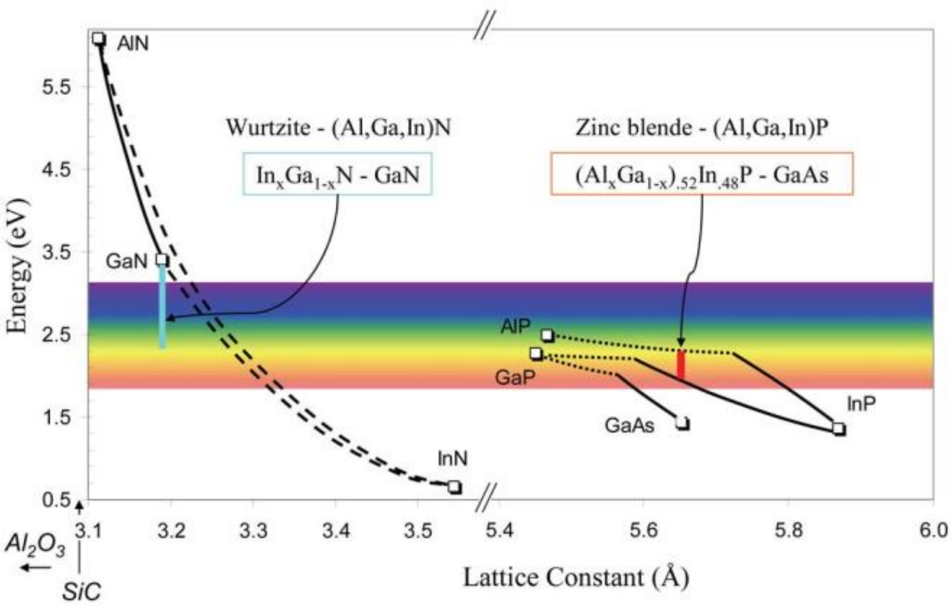
$$r_{spon,QW} = C_0\left(\frac{n^2\omega^2}{\pi^2\hbar c^2}\right)\frac{1}{L_z}\sum_{hh,lh}\sum_{n,m}\left|I_{hm}^{en}\right|^2 |\hat{e}\cdot\mathbf{p}_{cv}|^2\ \rho_{r,QW} H(\hbar\omega - E_{hm}^{en})\, f_c(\hbar\omega - E_{hm}^{en})(1 - f_v(\hbar\omega - E_{hm}^{en}))$$

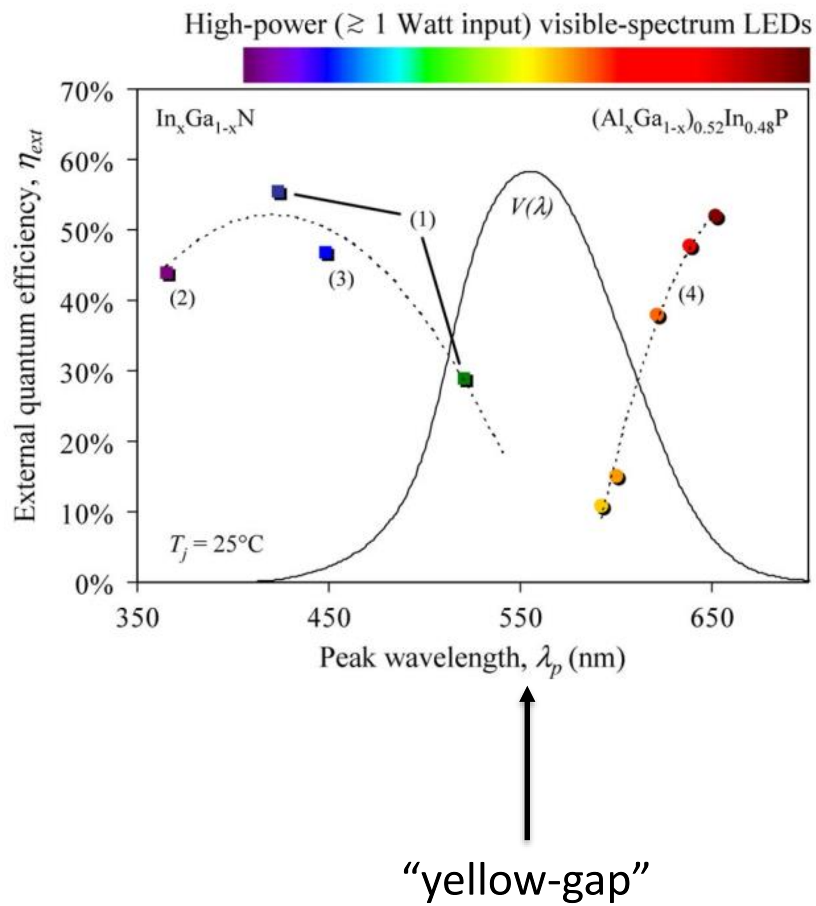


InGaAs/InP quantum well
6nm thick, T=300K

Note: Linewidth effects are ignored

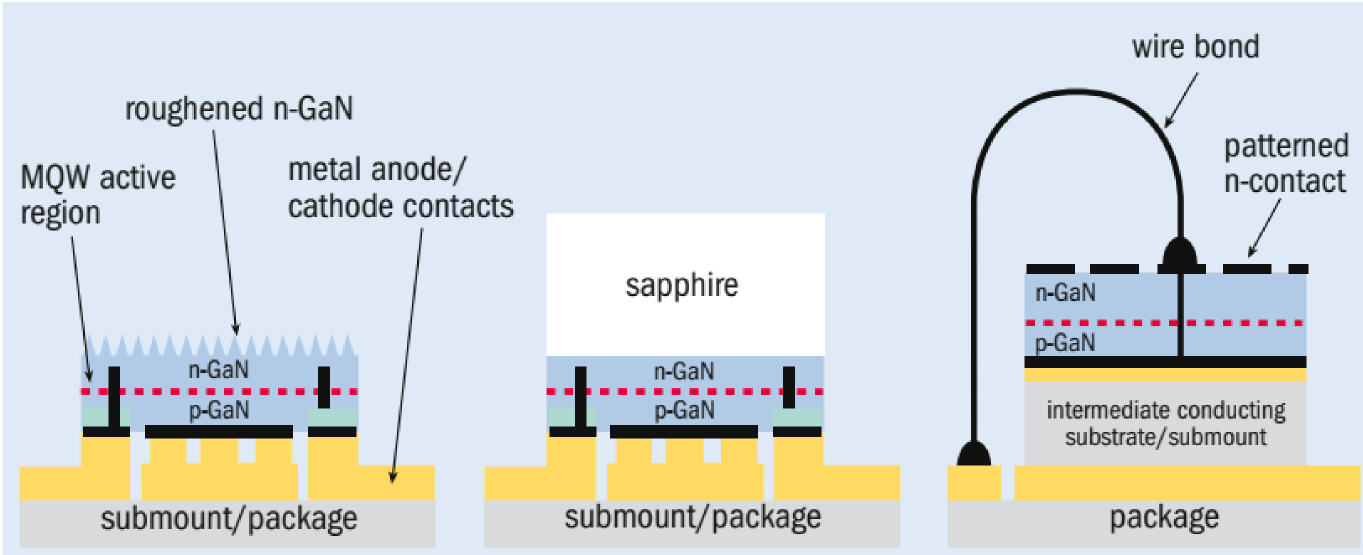
Light emitting diodes



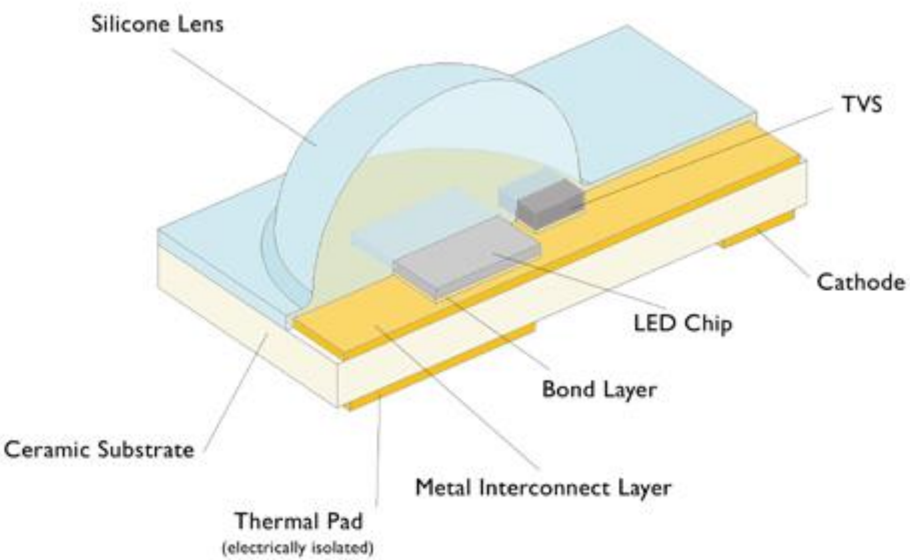


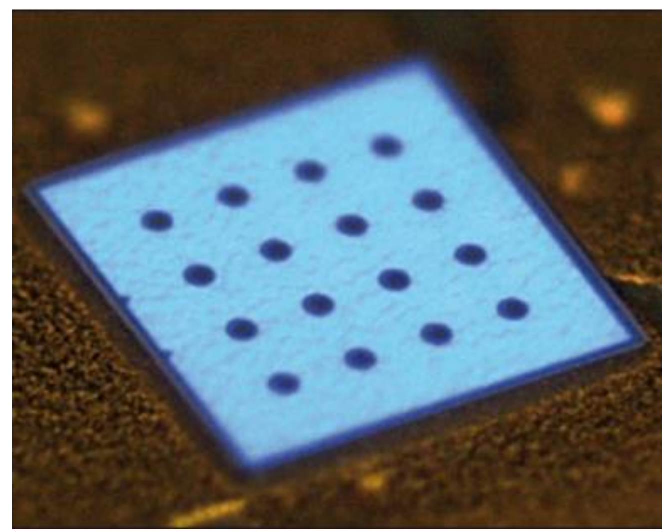
Krames et al. Journal of Display Technology, Vol. 3, No. 2, June 2007

Thin-film flip chip   flip chip   vertically-injected thin film



Source: Compound Semiconductor





Source: Lumileds